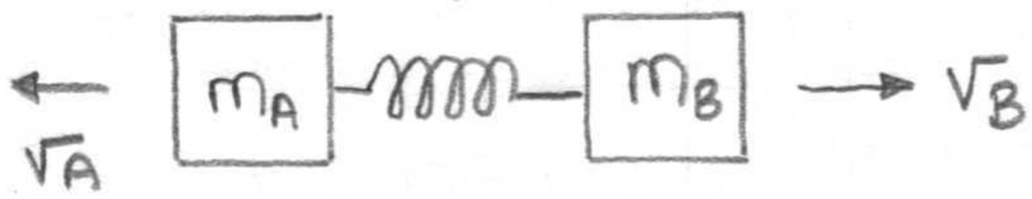
TAM 203
Homework Solutions

Due 2/19/08

9.74

spring compressed a distance δ and then released



Conservation of energy:

$$PE_{spring} = \frac{1}{2} k \delta^2$$

$$KE_{blocks} = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$\therefore \quad k\delta^2 = m_A v_A^2 + m_B v_B^2$$

Conservation of momentum:

$$m_A v_A - m_B v_B = 0 \qquad \therefore \quad v_B = \frac{m_A}{m_B} v_A$$

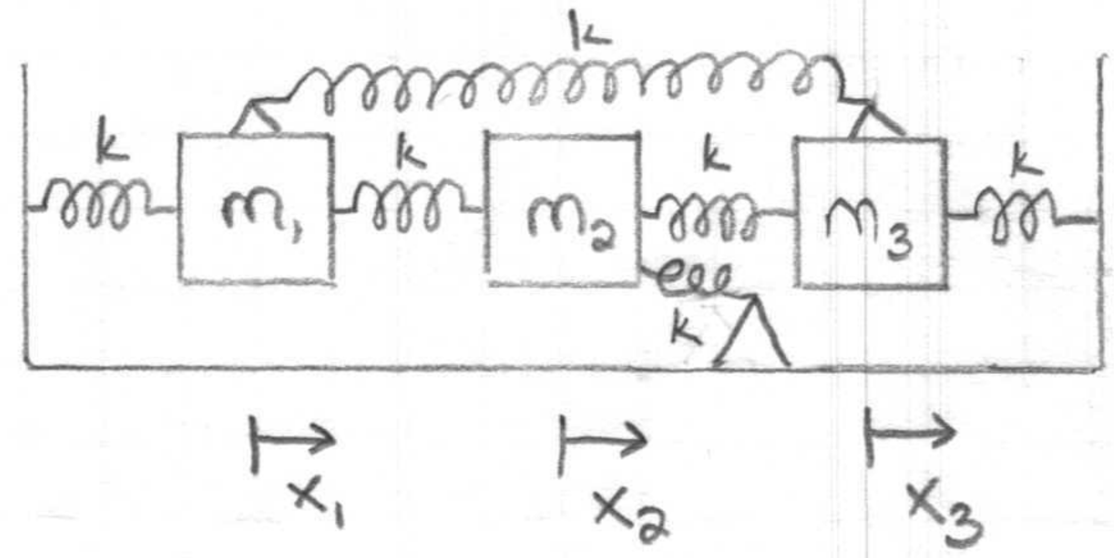
$$k\delta^2 = m_A v_A^2 + m_B \left[\frac{m_A}{m_B} v_A\right]^2$$

$$= m_A v_A^2 + \frac{m_A^2 m_B}{m_B^2} v_A^2 = m_A v_A^2 \left(1 + \frac{m_A}{m_B}\right)$$

$$= \frac{m_A + m_B}{m_B}$$

Solving for v_A, $\boxed{v_A = \sqrt{\frac{m_B k \delta^2}{m_A (m_B + m_A)}}}$

9.81

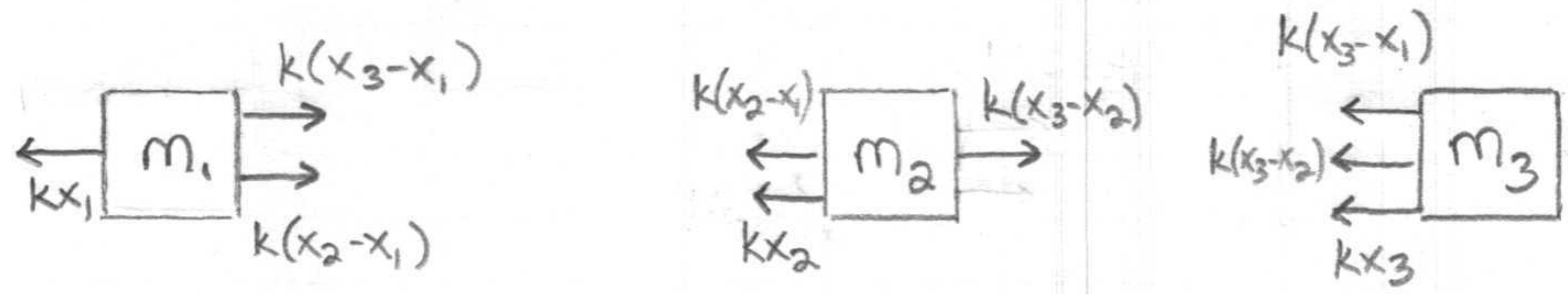


$m_1 = m_2 = m_3 = m = 1\ \text{kg}$

$k = 1\ \text{N/m}$

a)

Free body diagrams:



Equations of motion:

$$m\ddot{x}_1 = kx_3 - kx_1 + kx_2 - kx_1 - kx_1 = k(-3x_1 + x_2 + x_3) \quad (1)$$

$$m\ddot{x}_2 = kx_3 - kx_2 - kx_2 + kx_1 - kx_2 = k(x_1 - 3x_2 + x_3) \quad (2)$$

$$m\ddot{x}_3 = -kx_3 + kx_1 - kx_3 + kx_2 - kx_3 = k(x_1 + x_2 - 3x_3) \quad (3)$$

$$\therefore \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} = \frac{k}{m}\begin{bmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \text{ assume } \vec{x} = \vec{v}\sin(\lambda_n t)$$

$$\therefore -\lambda_n^2 \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \frac{k}{m}\begin{bmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \text{ which is in}$$

the form $[A]\{v\} = \lambda\{v\}$, where $\lambda = \lambda_n^2$

and $[A] = \dfrac{-k}{m}\begin{bmatrix} -3 & 1 & 1 \\ 1 & -3 & 1 \\ 1 & 1 & -3 \end{bmatrix}$, where $k = m = 1$

Use Matlab to find the eigenvalues and eigenvectors.

[v l] = eig([A]). See results on next page.

From Matlab,

$\lambda_1 = -4 \qquad v_1 = [-0.8018 \quad 0.2673 \quad 0.5345]^T$

$\lambda_2 = -4 \qquad v_2 = [-0.1543 \quad 0.7715 \quad -0.6172]^T$

$\lambda_3 = -1 \qquad v_3 = [0.5774 \quad 0.5774 \quad 0.5774]^T$

This indicates that as long as we set our initial conditions in proportion with these mode shapes (v_1, v_2, v_3), the system will vibrate with the same frequency. There are therefore 3 unique initial conditions for which normal mode vibrations result.

b) $m\ddot{x}_2 = k(x_1 - 3x_2 + x_3)$, $\{x_0\} = [0.1 \quad 0 \quad 0]$

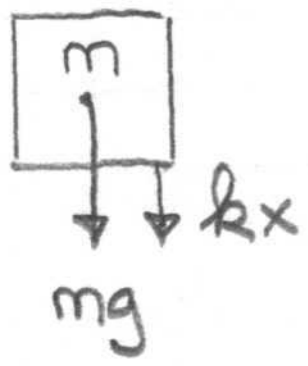
$$\therefore \quad \ddot{x}_2 = \frac{k}{m}(0.1 - 0 + 0) = (0.1\,m)\,\frac{1\ N/m}{1\ kg} = \boxed{0.1\ m/s^2}$$

9.83

a) Static equilibrium occurs when $mg = -kx$

$$\boxed{x = -mg/k}$$

b) FBD:

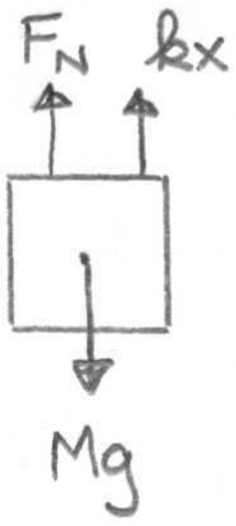


$$\Sigma F = ma$$

$$mg + kx = -m\ddot{x}$$

OR $$\boxed{\ddot{x} + \frac{k}{m}x = -g}$$

c) FBD:



d) mass leaves ground when $F_N = 0$

$\therefore \; Mg = kx \Rightarrow \boxed{x = Mg/k}$

e) When mass M is in the air, the stretching of the spring is $x-y$.

$$\therefore \; \boxed{\begin{aligned} \ddot{x} + k/m\,(x-y) &= -g \\ \ddot{y} + k/M\,(x-y) &= g \end{aligned}}$$

f) Same as (d), no value for $x < 0$.

Matlab codes will vary - try playing around. See website for an interesting paper written on this topic.